A market consistent framework for the fair evaluation of insurance contracts under Solvency II

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Abstract

The entry into force of the Solvency II regulatory regime is pushing insurance companies in engaging into market consistence evaluation of their balance sheet, including the financial options and guarantees embedded in life with-profit funds. The robustness of these valuations is crucial for insurance companies in order to produce sound estimates and good risk management strategies, in particular for liability driven products such as with-profit saving and pension funds. This paper introduces a simulation approach for Monte Carlo evaluation of insurance assets and liabilities, which is more suitable for risk management of liability driven products than common approaches generally adopted by insurance companies, in particular with respect to the assessment of valuation risk.

Keywords: Solvency II, economic scenario generator, minimum guaranteed option, sensitivity analysis

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1 Introduction

During the last decade, the European Community, in particular the European Insurance and Occupational Pension Authority (EIOPA), and the International Accounting Standards Board (IASB) have introduced new standards for insurance balance evaluation with the goal of establishing and maintaining compatibility between accounting and regulatory standards for harmonization of requirements, transparency and avoidance of arbitrage, both among jurisdictions and across other financial sectors such as banking. This effort has led to the Solvency II directive [Commission, 2015] and the IFRS 17 (Insurance Contract standards, [IASB, 2017]), which are respectively, already entered into force or will enter into force soon[1]. Parallel with the development of the Solvency II and IFRS principles has been the evolution of the rules for the voluntary publishing of embedded values by life companies. The CFO Forum, made up principally of European multinational representatives, continues to enhance these conventions, now dubbed Market Consistent Embedded Value (MCEV).

Like Solvency II and the IFRS principles, MCEV obviously, relies on market consistent valuation of assets and liabilities. These market-consistent evaluations are mostly carried out in practice by applying the Certainty Equivalent approach (CEQ) in discounting expected cash flows, which implies adjusting contractual cash flows for the implied risk premium above the risk-free rate provided by EIOPA (see [CFO-Forum, 2016a] and [EIOPA, 2017]). The paper [Gambaro et al., 2017] has shown how this approach can lead to biased results in the valuation of contractual financial options in traditional insurance products where the pay-off is determined by statutory accounting rules.

Another important aspect affecting market consistent valuations, is the rigid interpretation of market-consistency, which very often restricts the instruments to which stochastic models are calibrated to few categories of options such as interest rate swaps and equity indexes. An explanation of this common practice can be found in MCEV guidelines, where it is explicitly recommended the use of implicit volatilities from liquid options for the calibration of stochastic models, although there is allowance for historical calibration of parameters, which, like correlation, cannot be derived otherwise (see [CFO-Forum, 2016a] and [CFO-Forum, 2016b], principle 15). Another reason is the belief that accurate and reliable expectations about the future financial market levels can be derived only from the most recent financial security prices.

The emphasis on (efficient) price discovery power of financial market, which is implicit in this strict interpretation of market-consistency, can be found also in Solvency II principles[2] and in International Accounting Standards[3].

[1] IFRS 17 is effective from 1 January 2021. A company can choose to apply IFRS 17 before that date, but only if it also applies IFRS 9 Financial Instruments and IFRS 15 Revenue from Contracts with Customers.The Board will support the implementation of IFRS 17 over the next three and half years.

[2] See definition (7) [Commission, 2015], insurance and reinsurance undertakings' valuation of the assets and liabilities using the market consistent valuation methods prescribed in international accounting standards adopted by the Commission in accordance with Regulation (EC) No 1606/2002, should follow a valuation hierarchy with quoted market prices in active markets for the same assets or liabilities being the default valuation method in order to ensure that assets and liabilities are valued at the amount for which they could be exchanged in the case of assets or transferred or settled in the case of liabilities between knowledgeable and willing parties in an arm's length transaction. This approach should be applied by undertakings regardless of whether international or other valuation methods follow a different valuation hierarchy.

[3] The estimates of future cash flows shall be current, explicit, unbiased, and reflect all the information available to the entity without undue cost and effort about the amount, timing and uncertainty of those

Moreover, for some risk factors as sovereign or corporate credit risk, liquid options are not available in all markets, hence it is also possible to have a flexible stock model that can make good use of the historical information available through financial markets. We present a prototypical scenario generator able to deal with many sources of risk (such as interest rate, sovereign, rating and sector spreads, default risk), and with a wider spectrum of applications. In fact, by specifying the dynamics of risk factors under the real world probability and under the risk neutral measure, it is possible to carry out either risk management analysis, or market consistent valuations, without changing the number of risk drivers involved. This gives a greater explanatory power to this approach especially with respect of valuation risk, a subject which cannot be tackled easily with the most used approach.

The proposed model is applied to the valuation of embedded options in life insurance with-profit contracts (Value Of Guarantees, or VOG). As explained in details in [Gambaro et al., 2017], a with-profit (or participating) policy is a life insurance contracts offering certain guarantees to the policyholder, as a minimum rate of return. The policy is written on a segregated fund that is owned by the life insurance company, and that must be kept separate from the company's other assets. These funds consist of a pool of investments in securities such as bonds and stocks but their value does not fluctuates according to the market value of the underlying securities for different reasons. Firstly, the fund returns are estimated using accounting rules different from the fair value IFRS rules. Similar accounting rules are applied to traditional saving and pension products of different countries in continental Europe (Italy, Spain, France, Germany). Moreover, the determination of the return of the segregated fund is subject to discretionary rules (or management actions) applied by the insurance company, for instance the investment strategy or bonus mechanism (smoothing and market value reduction).

Considering the big market share of such products in many countries, the analysis of traditional life insurance contracts is a very important topic[4]. A non comprehensive list of the related literature is [Briys and De-Varenne, 1997], [Bacinello, 2001], [Bacinello, 2003], [Jϕrgensen, 2004] and [Bauer et al., 2006]. The existing literature analyse different aspects of the no-arbitrage fair evaluation of with-profit life insurance products as bonus mechanism, policy-holder surrender option, mortality risk However, the existing literature has some limitations: simple models for the dynamic of the segregated fund (few risk factors, for instance an Hull and White model for the short rate and a geometric Brownian motion for the fund value), the fund dynamic is modelled only under the pricing probability measure, hence, a link between pricing and capital requirement calculation of the liabilities is missing, the impact of accounting rules on the contract valuation is not considered, the role of asset allocation and investment strategies is not analysed, and financial calibration issues, that are fundamental for market consistency, are ignored.

In the insurance industry, a common practice adopted in market consistent valuation is to apply the Certainty Equivalent approach (CEQ), which consists in discounting the cash flows using a regulatory risk-free interest rate curve ([EIOPA, 2017]) and in a deterministic risk adjustment of the cash flows for credit and liquidity risks.

In the valuation of options embedded in with-profit policies, we test the impact of the common practice (CEQ) against our proposed model, the accounting rules and different investment strategies. These results are especially relevant because the CEQ is an indus-

future cash flows. They should reflect the perspective of the entity, provided that the estimates of any relevant market variables are consistent with observable market prices. [IFRS 17:33, Measurement]

[4]See data on life insurance market at https://www.insuranceeurope.eu/insurancedata.

try standard in the estimation of VOG. In particular, we will show how adopting a more general framework for this kind of valuations it is possible to retain all the nice features of the CEQ (namely market consistency and no arbitrage), while avoiding the reliance on few risk factors (mainly interest rate and equity).

The paper is structured as follows. In section 2 we briefly expose some applications of real world and risk neutral models to insurance products and discuss the motivations for a newer approach, focussing also on the implication on risk management. In section 3, we present our model and in section 4 we describe the calibration procedure and the challenges of calibrating it to real data. In particular, we estimate the probability distributions of the parameter estimators and test their standard errors using different approaches in order to provide robustness to the estimates.

In section 5 we show the application of our model to the evaluation of financial options in life insurance with-profit contracts.

Finally, in section 6 the sensitivity analysis of the option value is performed with respect to the different source of risk in the underlying fund and we estimate the impact on option price of the statistical uncertainty of calibrated parameter values (valuation risk), using a set of metrics which are not possible under the classical market consistent paradigm on which the CEQ approach hinges.

2 Proposal for a market consistent economic scenario generator.

In this section we discuss the motivations behind our proposal of a market consistent economic scenario generator. In particular we will concentrate on real world and risk neutral applications in Monte Carlo simulations because their relevance in financial and actuarial valuations has grown exponentially in the recent years, due to the increased availability of calculation power made disposable by the diffusion of cloud computing.

Although a treatment in their generality of real world, or risk neutral modelling is certainly out of the scope of this work, it is important to consider here some of their main characteristics because we think it's very important to establish a link between the two in order to adequately manage risks, including valuation risk, by insurance companies.

The demand for Economic Scenario Generators (ESG), a computer-based model of an economic environment that is used to produce the simulations of financial and economic variables used in Monte Carlo models[5], has grown due to the increasing complexity of financial and insurance products, which cannot be evaluated satisfactory well by analytical techniques, and due to the increasing demand of analysis for regulatory and administrative purposes. Generally speaking, we can say that real world (RW) and risk neutral (RN) models are used primarily in two applications: for risk management and solvency capital requirement (SCR) assessment (RW) and for market-consistent valuations such as the pricing of insurance products with embedded options and guaranties (RN). In particular the SCR valuation requires the integrated use of both the probability measures, in fact SCR consists on a risk measure calculated on the real world distribution of a one-year projected risk neutral (and market consistent) valuation of assets and liabilities. Real world models are increasingly used also for multi-period Strategic Asset Allocation (SAA)

[5] For a comprehensive introduction to ESGs and their applications to insurance and pension funds, see [SOA, 2016]

by long-term investors, such as insurance companies, or pension funds. This latter application is particularly important for liability-driven investment managers[6] seeking the stability of cash flows to fund liabilities, or the enterprise value maximisation.
The fundamental distinction between the two modelling approach follows directly from the differences in application purposes. For real world models the main objective is to find forecasts of risk and return over the relevant investment horizon, which are consistent with stylized facts observed in financial markets and economies. For risk neutralmodels instead, the objective is to create a pricing framework, free of arbitrage opportunities between risky securities and the risk free security used as a numeraire[7], which is able to provide valuations consistent with the (observed) market price of securities.
The principles behind Market-Consistent valuations derive mainly from risk neutral pricing. It seems that over time, a common interpretation of these has prevailed in the insurance sector, in particular concerning the choice of calibration instruments, which in many cases is restricted to interest rate swaps (IRS), swaptions and equity index options, and the amount of historical information used in valuations, which is generally one day of trading prices (usually December 31st). We think this interpretation is stricter than necessary and potentially counter productive. In fact, as noted by [Karoui et al., 2015], among others, the choice of using the information from a single day of trading to determine the Solvency Required Capital, makes this valuation exposed to the risk of price manipulation. Moreover, not all the risk factors which are relevant for describing the risks to which an insurance product or company are exposed, and which are considered in real world valuations, can be modelled considering only liquid options on one specific trading day. Particularly important in this respect are correlation, credit, liquidity and model risk, which can be assessed better using (also) historical information. Eventually, credit and liquidity risks are very important for insurance companies since the basis between European sovereign bonds and swap rates emerging during liquidity crisis (when for example, a flight-to-quality makes some European treasury bonds trading with larger bid-ask spread than other), can impair hedging strategies based on swaptions and the natural offsetting ability of liabilities to contrast interest rate movements[8]. This was recognised by the European regulator which, after the credit crunch in 2008, has introduced a series of counter cyclical measures, which have lead to the current Volatility Adjustment, a sort of (il)liquidity spread that can be added to the risk free rate before discounting liability cash flows[9]. It is remarkable that the Volatility and also the Matching Adjustment, are derived using historical information on securities, in other words not relying only on the information available on a single day of trading, as it is conventionally done for calibrating interest rate and equity models.

[6] By and large, liability-driven investments are saving or pension products, like segregated fund, where the way assets performance affects liabilities is critical for the sustainability and success of the investment strategy.

[7] For the definition of numeraire, Equivalent Martingale Measure and the Fundamental Theorem of Asset Pricing, see for example [Harrison and Pliska, 1981] and also [Black and Scholes, 1973] as a good introduction to Risk Neutral pricing and to the role played by the no arbitrage assumption, among the others, in deriving their famous valuation formula.

[8] In market-consistent evaluations liability cash flows are discounted using a risk free curve derived from 6 months euribor, which is constructed as prescribed by EIOPA. After the financial crisis in 2008, some European sovereign bond issuers, the so called PIGS (Portugal, Italy, Greece and Spain) began to trade with a material spread over euribor. Therefore, the assets of many insurance companies began to deteriorate while liabilities didn't due to the basis, or liquidity effect between market prices and discounting factors used to assess the economic value of technical provisions.

[9] See [EIOPA, 2017]

Finally, it is difficult to derive a model for corporate bonds inclusive of rating migration and default cost estimates, purely on quoted options and without relying on historical information at all[10].
Considering that in many important valuations that insurance company are asked to perform (simply consider Solvency Capital Requirement, or ORSA[11]), real world and risk neutral models have to be used together, we think it is crucial to reduce the distance in terms of the number of risk factors adopted by the two models. Ultimately, a more flexible and richer approach to market-consistent modelling, could lead to a greater ability to measure and manage the different types of risks an insurance company is exposed to, including valuation risk, which is currently almost neglected[12].
Instead, we propose to consider an alternative way to calibrate and build financial stochastic models which allows for greater flexibility without bringing tantamount complexity. Among the advantages of our modelling approach is that the calibration of model parameters on historical series allows naturally a model specification under the real world and the risk neutral measure, giving a wider perspective to our model both for risk management and pricing. Ultimately, the ability of an ESG to produce consistent scenarios in the real world and in the risk neutral probability measures is a desirables feature for practitioners as highlighted for instance in the 2016 ESG practical guide of the Society of Actuaries ([SOA, 2016]).
Moreover, our ESG is market consistent in the sense of [Kemp, 2009]. In fact, it allows under the risk neutral measure the perfect fitting of term structure of the risk free zero coupon bond (ZCB) prices and of sovereign and corporate bond prices curve at a fixed date. Obviously, the scenarios produced for risk neutral evaluations satisfy the martingale restrictions, in the sense that all securities priced over the simulated risk factors show performances which are martingale with respect to the asset chosen as numeraire.
Finally, historical calibration makes possible to develop easily robustness test on the calibration results and sensitivity analysis in order to asses the calibration risk[13], and model risk, by making model choice a consequence of the trade-off between variance explained and complexity in model specification. These analysis are an innovation for ESG practice and they are of difficult, if not impossible, implementation using fixed date calibration (see 6).
In order to calibrate the parameters involved in the interest rate dynamic, it is a common

[10] see for example [A. Arvanitis, J. Gregory, J.P. Laurent, 1998]

[11] At the heart of the prudential Solvency II directive, the Own Risk and Solvency Assessment (ORSA) is defined as a set of processes constituting a tool for decision-making and strategic analysis. It aims to assess, in a continuous and prospective way, the overall solvency needs related to the specific risk profile of the insurance company.

[12] With Valuation Risk we mean correlation, basis, liquidity, and model risks in accordance with the prudent person principle as stated in the article 132 of Solvency II Directive, which quotes:

1. Member States shall ensure that insurance and reinsurance undertakings invest all their assets in accordance with the prudent person principle, as specified in paragraphs 2, 3 and 4.
2. With respect to the whole portfolio of assets, insurance and reinsurance undertakings shall only invest in assets and instruments whose risks the undertaking concerned can properly identify, measure, monitor, manage, control and report, and appropriately take into account in the assessment of its overall solvency needs in accordance with point (a) of the second sub-paragraph of Article 45(1)...

[13] We mean by calibration risk the impact that the uncertainty in the statistical estimation of a parameter can have on the probability distribution of risk factors and hence on the economic valuation.

practice to use the implied interest rate swaptions volatility cube. Our ESG can be modified in order to calibrate interest rates dynamic to swaptions volatility historical series, although we do not consider it strictly necessary.
In fact, in financial markets, the employment of certain market data for model calibration is justified with hedging purposes. For instance, the risk of a (fixed notional) callable bond can be hedged using a Bermuda swaption, hence a calibration to quoted swaption prices of the interest rate model is obviously desirable. Moreover, as already discussed in [Gambaro et al., 2017] the traditional with-profit insurance funds have a complex financial structure due to several factors such as accounting rules, management actions, insurance risks, as for instance the mortality risk, and a bonus rate mechanism that depends on the history of fund returns. Hence, hedging is an open problem in this setting (see [Luciano and Regis, 2014] for a discussion of hedging strategies in presence of mortality risk for a simplified insurance product). Some insurers buy simple financial instruments (e.g. swaptions, cap and floor) to hedge some part of their asset portfolios against financial movements, but this fact alone cannot be considered systematic hedging of the financial guarantees embedded in traditional with-profit products[14]. Also, hedging with options may be "impaired" by the accounting rules used for reporting a company's balance sheet, which may allow hedge accounting only under some required conditions[15], and by the rules used to calculate the insurance fund return (statutory, or mark to market)[16].

3 The model

The models for sovereign and corporate bonds discussed in this section allow for three different sources of risk: interest rates, credit and liquidity or sector based risk. The three risk factors are stochastic and a dependence between different sovereign issuers (or corporate sectors) is introduced. The aim of this section is to specify the dynamic of risk factors that affect sovereign and corporate bonds and to report the valuation formulae under the risk neutral measure.

3.1 Sovereign zero coupon bond pricing formula

At first, we model the risk free interest rate curve using the parsimonious and widely adopted Hull and White model (see [Hull and White, 1990]). The dynamics of the short rate $r(t)$ is described by the following stochastic differential equation

$$\begin{aligned} dx(t) &= -a\,x(t)dt + \sigma dW(t), \\ x(0) &= 0, \\ r(t) &= x(t) + \alpha(t) \end{aligned} \tag{3.1}$$

where a is the so called speed of reversion coefficient, σ is the volatility parameter and $W(t)$ is a standard Brownian motion. The adoption of a Gaussian interest rate model is consistent with the recent experience of negative rates in the Euro zone. Moreover, the deterministic function of time $\alpha(t)$ allows a perfect fitting of the initial term structure of risk free zero coupon bond prices and it is equivalent to assume a deterministic and time

[14] For an overview of insurance participating contracts see [Pitacco, 2012].
[15] See for example the hedging requirement under IFRS 9 financial instruments
[16] We comment further on swaptions in the Appendix D.

dependent long run mean parameter in the Vasicek model. Without loss of generality, the above dynamic is assumed to hold under the risk neutral measure.
The time t price of a risk free Zero Coupon Bond (ZCB) with maturity T is obtained by computing the following expectation under the risk neutral measure

$$P(t,T) = \mathbb{E}_t\left[e^{-\int_t^T r(s)\,ds}\right],$$

and it can be easily shown (see [Brigo and Mercurio, 2006] page 75 or [Hull and White, 1990]) that this expectation can be written as

$$P(t,T) = \frac{P(0,T)}{P(0,t)} e^{A(t,T)-B(t,T)\,x(t)}, \tag{3.2}$$

where

$$\begin{aligned} B(t,T) &= \frac{1-e^{-a(T-t)}}{a}, \\ A(t,T) &= -\frac{\sigma^2}{4a}(1-e^{-2a\,t})B(t,T)^2 - \frac{\sigma^2}{2a^2}(1-e^{-a\,t})^2 B(t,T), \end{aligned}$$

and $P(0,T)$, $P(0,t)$ are the initial risk free ZCB market prices.
In order to model the price of a bond issued by a defaultable issuer, we adopt an intensity model with zero recovery. This is equivalent, see for example [Jarrow and Turnbull, 1995], to add a spread to the short rate. The spread is related to the creditworthiness of the issuer I. Therefore, the price of a defaultable ZCB is obtained as

$$P^I(t,T) = \mathbb{E}\left[e^{-\int_t^T (r(u)+s^I(u))\,du}\right]. \tag{3.3}$$

Assuming independence between spread and risk free short rate model, the price of the ZCB can be split into the product of two components, the risk free ZCB times the survival probability, $SP^I(t,T)$, i.e. the probability that the issuer does not default in the time interval $[t,T]$,

$$P^I(t,T) \;=\; P(t,T)\,SP^I(t,T),$$

where

$$SP^I(t,T) \;=\; \mathbb{E}\left[e^{-\int_t^T s^I(u)\,du}\right]. \tag{3.4}$$

The credit spread $s^I(t)$ is modelled as a positive stochastic process. We adopt the so called CIR++ model, i.e. a square-root Cox-Ingersoll-Ross (CIR) process plus a deterministic adjustment in order to fit the initial survival probability term structure (see, [Cox et al., 1985], [Brigo and Mercurio, 2006] page 102). Therefore, we write

$$\begin{aligned} dy^I(t) &= b_I\left(\bar{s}_I - y^I(t)\right)dt + \eta_I\sqrt{y^I(t)}dZ_I(t), \\ y^I(0) &= y_0^I, \\ s^I(t) &= y^I(t) + \psi^I(t). \end{aligned} \tag{3.5}$$

where Z_I is a standard Brownian motion, assumed to be independent from the Brownian motion driving the dynamics of the risk-free rate and $\psi^I(t)$ is a deterministic function of

time that allows a perfect fitting of the initial term structure of specific issuer survival probabilities.
If the spread follows the dynamic in equation (3.5), then the survival probability (3.4) can be expressed in closed form (see [Brigo and Mercurio, 2006] page 103)

$$
\begin{aligned}
(3.6)\quad SP^I(t,T) &= \frac{SP^I(0,T)}{SP^I(0,t)}\, e^{\bar{A}_c^I(t,T)-B_c^I(t,T)\, y^I(t)}, \\
\bar{A}_c^I(t,T) &= A_c^I(t,T) - \left(A_c^I(0,T) - A_c^I(0,t)\right) + \left(B_c^I(0,T) - B_c^I(0,t)\right)\, y_0^I, \\
A_c^I(t,T) &= \frac{2b_I\bar{s}_I}{\eta_I^2} \log\left[\frac{2h\, e^{(b_I+h)(T-t)/2}}{2h + (b_I+h)\left(e^{(T-t)\,h}-1\right)}\right], \\
B_c^I(t,T) &= \frac{2\left(e^{(T-t)\,h}-1\right)}{2h + (b_I+h)\left(e^{(T-t)\,h}-1\right)}, \\
h &= \sqrt{b_I^2 + 2\eta_I^2},
\end{aligned}
$$

$SP^I(0,T)$ and $SP^I(0,t)$ being the initial market survival probabilities.
In our model the liquidity risks is taken into account including a liquidity spread, $l^I(t)$, in the ZCB formula

$$
(3.7)\qquad P^I(t,T) = \mathbb{E}\left[e^{-\int_t^T (r(u)+s^I(u)-l^I(u))\, du}\right].
$$

In this case we do not require $l^I(t)$ to take only positive values and we can use again the Hull and White extended Vasicek model or an Ho and Lee process, if the mean reversion of the process is not evident from market data. In case we use a Hull-White model the liquidity spread dynamic is

$$
(3.8)\qquad dz^I(t) = k_I(\bar{l}_I - z^I(t))dt + \phi_I dY^I(t),
$$

or in case we use a Ho-Lee model the dynamic is

$$
(3.9)\qquad dz^I(t) = \mu_I\, dt + \phi_I dY^I(t),
$$

in both cases we set

$$
\begin{aligned}
z^I(0) &= 0, \\
l^I(t) &= z^I(t) + \varphi^I(t),
\end{aligned}
$$

where Y^I is a standard Brownian motion, assumed to be independent from the Brownian motions driving the dynamics of the risk-free rate and the credit spread and $\varphi^I(t)$ is a deterministic function of time that allows a perfect fitting of the initial ZCB prices of the specific issuer.
As highlighted in [Gambaro et al., 2017], the possibility of having a negative liquidity spread is relevant for capturing the so called fly-to-liquidity effects. If we assume that the liquidity spread is independent from both the risk free rate and the credit spread, then, simply, the ZCB formula contains a second multiplicative adjustment factor in closed form.
The assumption of independence between the risk free rate, the credit and liquidity spread ensures the analytical tractability of the model and keeps it parsimonious. Other authors (see [Monfort and Renne, 2014]) consider more complex approach to correlate credit and

liquidity. We have preferred a yield decomposition like in [F. Longstaff and Neis, 2005], which has the advantage to be easier to calibrate using the so called cascade approach, while leaving the relative liquidity between securities of the same kind (like European sovereign bonds) free to vary.
Finally, credit and liquidity spreads of different issuers can be correlated through the Brownian motions of the credit or liquidity spread processes, i.e. for $I \neq J$

$$dZ_I(t)\, dZ_J(t) = \rho_c^{IJ}\, dt,$$
$$dY_I(t)\, dY_J(t) = \rho_l^{IJ}\, dt.$$

3.2 Corporate zero coupon bond pricing formula

The credit rating transition and the default process are modelled using an extension of the classical time-homogenous Markov chain. This model is presented for the first time in [Lando, 1998]. However, for a better mathematical and financial formulation of the model, the dynamic of the credit rating of a corporate bond, $Y(t)$, is presented using a stochastic time. $Y(t)$ is a process on a finite state space $N = \{1, 2, \ldots, K\}$, where K is the absorbing state of default and it is characterized by the transition matrix

$$P(s,t) = e^{A\,(\tau(t)-\tau(s))} \tag{3.10}$$

where A is a matrix with non-negative off-diagonal elements and zero row sums, i.e. A is the generator matrix of a time-homogeneous Markovian chain, and $\tau(t)$ is a stochastic time. Hence, conditionally to the trajectory of τ, Y is a (inhomogeneous) Markov process. Compared to [Lando, 1998], this formulation allows to easily express the conditions under which the model is consistent (e.g transition probabilities between 0 and 1) and the process Y is unconditionally Markovian.
In order for the model to be consistent, the process τ has to be a stochastic time, i.e. τ is a real positive and increasing right continuous process with left limits (RCLL), for every $t \geq 0$, $\tau(t)$ is a stopping time, $\tau(t)$ is finite almost surely, $\tau(0) = 0$ and $\lim_{t\to\infty} \tau(t) = \infty$ (see [Barndorff-Nielsen and Shiryaev, 2010] for a complete discussion). Moreover, as demonstrated in [Feller, 1971], if $\tau(t)$ has stationary (non-negative) independent increments, then Y is unconditionally a Markov chain and is called a subordinated process (τ is a subordinator).
The financial interpretation of the model is clear: the transition probabilities of corporate bonds belonging to the same market sector are subjected to a common source of randomness, i.e. the process τ.
If we assume that the matrix A is diagonalizable ($A = BDB^{-1}$, $D = \mathrm{diag}(d_1, ..., d_{K-1}, 0)$), then the transition matrix can be written as

$$P(s,t) = Be^{D\,(\tau(t)-\tau(s))}B^{-1}.$$

If we define $\tau(t)$ as an integral of a positive stochastic intensity $\lambda(t)$, i. e.

$$\tau(t) = \int_0^t \lambda(s)ds,$$

then the Kolmogorov's backward equation assumes the following form

$$\frac{\partial P(s,t)}{\partial s} = -A\,\lambda(s)\,P(s,t).$$

Hence, the generator matrix of Y is $A_\lambda(t) = A\,\lambda(t)$, this means that the instantaneous transition probability from the rating i to the rating j is $a_{ij}\,\lambda(t)dt$. The price of a zero coupon bond (ZCB) with maturity T and rating i at time t is

$$\begin{aligned} C_i(t,T) &= \mathbb{E}_t\left[e^{-\int_t^T r(s)\,ds}(1 - P(t,T)_{i,K})\right] \\ &= \sum_{j=1}^{K-1} -b_{ij}\,b_{jK}^{-1}\,\mathbb{E}_t\left[e^{-\int_t^T r(s)\,ds}e^{d_j\,\int_t^T \lambda(s)\,ds}\right], \end{aligned}$$

where $r(t)$ is the risk free short rate, $P(t,T)_{i,K}$ is the transition probability from a credit rating i at time t to a default state at time T, b_{ij} and b_{ij}^{-1} are the (i,j) elements of the matrices B and B^{-1}, respectively. If r and λ are independent, then we obtain

$$C_i(t,T) = C_0(t,T)\sum_{j=1}^{K-1} -b_{ij}\,b_{jK}^{-1}\,\mathbb{E}_t\left[e^{d_j\,\int_t^T \lambda(s)\,ds}\right],$$

where $C_0(t;T)$ is the price of a risk free ZCB. The process λ is modelled as an affine process, for instance a CIR++ model

$$\begin{aligned} dy(t) &= b\left(\bar{\lambda} - y(t)\right)dt + \eta\sqrt{y(t)}\,dB(t), \\ y(0) &= y_0, \\ \lambda(t) &= \phi(t) + y(t), \end{aligned} \tag{3.11}$$

where $B(t)$ is a standard Brownian motion and $\phi(t)$ is a positive deterministic function. A limitation of our model is that the function $\phi(t)$ allows a perfect fitting of the initial term structure for a chosen rating class, then the initial curves of other rating classes are not perfectly fitted. The expectation $\mathbb{E}_t\left[e^{d_j\,\int_t^T \lambda(s)\,ds}\right]$ has an analytical expression as in formula (3.6). Therefore, given the initial firm rating, this model is able to reproduce the (risky) ZCB term structure.
A more flexible dependence structure among the market sectors can be introduced adding liquidity spreads to the ZCB formula, i.e.

$$C_i^S(t,T) = C_0(t,T)\mathbb{E}\left[e^{-\int_t^T l^S(s)\,ds}\right]\sum_{j=1}^{K-1} -b_{ij}\,b_{jK}^{-1}\,\mathbb{E}_t\left[e^{d_j\,\int_t^T \lambda(s)\,ds}\right],$$

where S is the sector index and $l^S(t)$ is a stochastic process as in equation (3.8) or (3.9). The different sectorial spreads are then linked through a correlation matrix.

3.3 The real world dynamic of the risk factors

As previously discussed the specification of our model under the real world probability measure is important different reasons: risk management, the calculation of Solvency capital requirement and the estimation of the model parameters using historical data. In particular, the tractability of the risk factors dynamics under both real world and risk neutral measures gives an important advantage for risk management since then the historical information on risk drivers can be used to derive meaningful alternative calibration which can be used for stress testing. For example, a time interval including credit crunch or, going more back in time, the dot-com bubble, could be used to calibrate the

model and get a price of the contingent claims conditioned to the alternative stressed conditions.
Hence, we need to consider a functional specification for the real world model, which has a consistent (and convenient) specification in the risk-neutral framework. Therefore, for risk factors that follow an Hull and White (or a Ho and Lee) process under the risk neutral measure, we assume a Vasicek process (or a Brownian process with drift) under the real world measure. This choice is equivalent to assume a deterministic but time dependent market price of risk (to get the perfect fit on initial term structure). For instance, the dynamic of the risk free rate under the real world probability measure is

$$\begin{aligned} dr(t) &= a(\bar{r} - r(t))dt + \sigma dW(t), \\ r(0) &= r_0, \end{aligned} \tag{3.12}$$

where $\bar{r}$ and a are the so called long run mean and speed of reversion coefficient, σ is the volatility parameter and $W(t)$ is a standard Brownian motion under the real world measure. Therefore, we assume a deterministic but time dependent market price of risk (to ensure the perfect fit of initial term structure) for the risk free, the sovereign liquidity spread and the corporate sector-based risk factor. The detailed derivation is given in Appendix B.
Similarly, since the sovereign credit spread $s^I(t)$ and the rating transition intensity $\lambda(t)$ follows a CIR process plus a deterministic shift under the risk neutral measure (CIR++ model), we assume a CIR processes under the real world measure. For instance, the dynamic of the rating transition intensity process $\lambda(t)$ under the real world probability measure is

$$\begin{aligned} d\lambda(t) &= b\left(\bar{\lambda} - \lambda(t)\right)dt + \eta\sqrt{\lambda(t)}\, dB(t), \\ \lambda(0) &= \lambda_0, \end{aligned} \tag{3.13}$$

where $B(t)$ is a standard Brownian motion under the real world measure. The detailed derivation is in Appendix B.

4 Calibration

In this section we discuss the calibration procedure of the model described in the previous one.
According to the recent financial literature (for instance [Morini, 2009] and [Moreni and Pallavicini, 2014]), we identify the overnight rate (in particular the Eonia rate for Euro currency) to be the best proxy for the risk free interest rate. Hence, the short risk free rate $r(t)$ is calibrated on the historical series of the Eonia rate using a maximum likelihood method. The credit spreads of sovereign bonds are calibrated on historical series of default probabilities bootstrapped from sovereign (Italian and German) CDS spread.
The rating transition intensity process is calibrated on historical series of corporate default probabilities obtained via bootstrap of the Itraxx Europe CDS Index spread. We assume a reference rating for the index through analysis of its constituents. This is an important choice considering that our model is driven by one factor and it will not produce the same calibration for, let's say, a AA, or a BBB bond. However, since the CIR parameters of our corporate credit spread process are estimated through a maximum likelihood applied to the historical series of default probabilities (the sovereign credit model as well), it is possible to use different series, with different reference rating, and calibrate them jointly.

The relation between the observed data and the unobservable credit spread stochastic process is non linear, but it is monotonic and so invertible. Hence, we apply the likelihood method proposed in [Pearson and Sun, 1994], explained in details in Appendix A.
The sectoral adjustment factors are calibrated on EUR bonds sectorial indexes available in Bloomberg. We consider three sectors, i.e. industrial, financial and communications (tickers Bloomberg are BERCIN, BERCFI and BERCCO). In order to obtain the sectorial spread, we subtract to the index bond yield the risk free zero rate and the fundamental spread related to the rating quality of the index, i.e. the logarithm of the survival probability. The correlation matrix is obtained using historical series of spread daily returns.
Finally, the generator matrix is initialised using the historical transition matrices published by rating agencies. We suggest to avoid the utilisation of one year transition probability matrices if many elements are equal to zero because sparse matrices are neither consistent from a theoretical point of view and makes difficult the calibration on market data. A better approach is to consider five years transition matrix and rescale them to provide the (implied) one year probability.

4.1 Numerical results of calibration

In this section we present a possible complete calibration of the previously proposed model.
We choose the historical period from 20^{th} September 2016 to 13^{th} January 2017 for two reasons. The first motivation is the evident regime switching shown by the Eonia historical series (see Figure 1) in the last five years, and the fact that data are stable only after April 2016. The histogram in Figure 2 confirms the bimodal distribution of the Eonia rate in the last five years (from January 2012 to January 2017). The second reason is the length of the Itraxx Europe CDS Index series because for this exercise, we wanted to avoid discontinuities in the (average) rating quality of the data, given our limited access to publicly available (i.e. free of charge) corporate bond indexes CDS spreads at a daily frequency.
Maximum likelihood estimates of parameters under the real world measure are given in Tables 1-3. Figures 1, 3, 4 and 5 show that historical series agree with the model simulations. Figure 6 illustrates that the calibrated model for corporate credit risk is able to fit the historical series of market survival probabilities also for maturities different from the 5 years reference maturity.
We estimate the confidence intervals for the parameters estimators of the Cox-Ingersoll and Ross model using the bootstrapping technique described in [Efron and Tibshirani, 1986]. Using calibrated parameters in Tables 2-3, we simulate five thousands Monte Carlo paths and for each path we perform a new maximum likelihood calibration. In this way we build density histograms for the parameters estimators and we calculate the 95% confidence intervals. This procedure has three purposes. The first is testing the robustness of the calibration algorithm. In fact, when the convergence of the optimization algorithm is difficult, the density histograms show a bimodal (or multi-model) behaviour due to the presence of local minima in the optimization procedure. The second is to provide confidence bands for the estimated parameters when the Fisher information matrix is not invertible, or in presence of restrictions on parameters (and the CIR model is a case of the latter). The third is using the parameters density histogram in the sensitivity analysis, presented in Section 6.

Looking at sovereign and corporate sectoral bonds market data, we choose a Ho-Lee model under the risk neutral measure (and a Brownian motion with drift in the real world measure) for sovereign liquidity spreads and sector-based corporate spreads, hence only volatilities and historical correlations are estimated. In fact, the hypothesis of mean-reversion is not confirmed for these historical series because the speed of mean reversion parameter is not statistically different from zero. This is evidently an advantage provided by the use of historical series. We couldn't have come to this conclusion using cross-sectional data. Therefore, the exclusive use of this latter type of information in a model calibration, is prone to produce unfathomable model error.

a	$\bar{r}$	σ
4.2141 (0.0367)	-0.0408 (0.0018)	0.6073 (0.0003)
3.6551 (0.5112)	-0.3516 (0.0954)	0.0896 (0.0066)

Table 1: Risk free rate model: parameters of Vasicek model calibrated on the Eonia rate (in percentage) historical series from 2^{nd} January 2012 to 13^{th} January 2017 and from 20^{th} September 2016 to 13^{th} January 2017 for the first and the second row, respectively. The results of the calibrations are shown in Figure 1. Standard errors obtained with the Fisher information are reported in parenthesis.

Country	b	$\bar{s}$	η
GER	0.2199 (0.0068)	0.0037 (0.00003)	0.0404 (0.0002)
ITA	0.5430 (0.0308)	0.0270 (0.0001)	0.1712 (0.0020)

Table 2: Sovereign credit risk model: parameters of CIR model calibrated on the 5Y sovereign historical series from 20^{th} September 2016 to 13^{th} January 2017. The result of the calibration is shown in Figure 3. Standard errors obtained with the simulative procedure previously explained are reported in parenthesis.

The historical correlation estimated using daily returns of Italian and German credit spreads from 20^{th} September 2016 to 13^{th} January 2017 is $\rho_c = 0.4222$. The 95% confidence interval is $[0.2591, 0.5618]$. The historical correlation estimated using daily returns of Italian and German liquidity spreads from 20^{th} September 2016 to 13^{th} January 2017 is $\rho_l = 0.6145$. The 95% confidence interval is $[0.4861, 0.7170]$.

b	$\bar{\lambda}$	η
0.9951 (0.0191)	3.0830 (0.0073)	1.0871 (0.0010)

Table 3: Transition matrix: parameters of CIR model calibrated on the 5Y Itraxx Europe CDS Index S26 historical series from 20^{th} September 2016 to 13^{th} January 2017. The result of the calibration is shown in Figures 5 and 6. Standard errors obtained with the bootstrap technique previously explained are reported in parenthesis.

The correlation matrix estimated using historical series of industrial, financial and communications sector spreads is

$$\rho = \begin{bmatrix} 1 & 0.9828 & 0.9799 \\ 0.9828 & 1 & 0.9682 \\ 0.9799 & 0.9682 & 1 \end{bmatrix}. \tag{4.1}$$

We also estimate lower and upper bounds of the correlation matrix which represent the 95% confidence interval, the upper bound is

$$\rho_{LB} = \begin{bmatrix} 1 & 0.9734 & 0.9691 \\ 0.9734 & 1 & 0.9511 \\ 0.9691 & 0.9511 & 1 \end{bmatrix},$$

and the lower bound is

$$\rho_{UB} = \begin{bmatrix} 1 & 0.9888 & 0.9870 \\ 0.9888 & 1 & 0.9793 \\ 0.9870 & 0.9793 & 1 \end{bmatrix}.$$

Corporate sectors turn out to be highly correlated, in order to understand this extreme feature in calibrated parameters, we perform a six months rolling window valuation of the correlations from August 2009 to January 2017 shown in Figure 7. We conclude that the high correlations between corporate sectors are a recent feature that characterizes the present market behaviour.

4.2 Martingale test

In order to prove that the Economic Scenario Generator (ESG) built upon the model presented in previous section is arbitrage free and market consistent, martingale tests on sovereign and corporate coupon bonds with different maturities are performed under the risk neutral measure, as explicitly required by the Solvency II directive,. The martingale process is built by dividing the total return performance of an asset by the total return performance of the cash account, i.e. the numeraire of the risk neutral measure, defined as

$$M(0,t) = e^{\int_0^t r(s)ds}$$

where $r(t)$ is the short rate in equation 3.1.

Particular attention has to be taken in presence of the liquidity spread. In fact the numeraire under which the total return index is a martingale is the cash account plus a correction for the liquidity basis. A similar problem is observed in multiple interest rate curve models in which the Forward Rate Agreement (FRA) rate is not a martingale with respect to the overnight ZCB as numeraire, see [Bianchetti, 2012] for a detailed discussion. Hence, similarly to [Bianchetti, 2012], we use the quanto adjustment commonly encountered in the pricing of cross-currency derivatives to build martingale indexes. The results are shown in Figure 8, using the calibrated parameters presented in Tables 1-3.
It is important to notice that affine processes allow to decompose the yield of a security in the following building blocks:

$$Y = R + S + L,$$

where R is the risk free rate, S is the credit spread (driven by the likelihood of default) and L is a liquidity spread (driven by agents' preferences on that security and by market micro structure). In order to obtain a martingale process, the price return process has to be adjusted by the surviving probability[17], and by the drift of the processes of L. Since

[17] $(1 - PD(0,t))$ where $PD(0,t)$ is the probability of a security defaulting between time 0 and t.

we have chosen to model L using a currency analogy, a quanto adjustment is the natural choice, but this is also a sensible one considering, as we did, that the most important driver of liquidity spreads are market participants' preferences and expectations, which affect the demand of a security.

5 Application to financial options embedded insurance products with minimum guaranteed return

In this section, we apply the propoed market consistent economic scenario generator to the valuation of contractual options embedded in Italian life insurance with-profit traditional products[18]
Results show that adjusting cash flows for risk as in the CEQ approach inadvertently affects the value of the option through the statutory accounting rules of the segregated fund. Moreover, we compare the value of guarantee obtained using the Italian accounting rules (LGAAP) and the mark to market rules and we note the great impact of the accounting rules on the valuation, which is ignored in the related literature. Finally, we test and compare two different investment strategy of the fund (buy & hold and constant mix).
In order to test our model and price insurance products options, we have set up a full ALM simulation based on a specifically engineered MATLAB® code, which adopts an approach derived from [Castellani et al., 2005]. Alternatively, it is possible to use other ALM softwares in commerce, which employ a similar methodology.

5.1 Description of the tested portfolio

The ALM set-up described in the previous section is used to simulate, over a 20 years time-horizon, a portfolio of endowments, i.e. life liabilities with death, surrender and maturity benefit (no annuities), which has a total duration (modified) of about nine years and which runs-off in approximately 20 years. Liabilities have an average minimum return guaranteed of 3% ($\bar{r}$), a total value (mathematical reserve) of one billion and a vintage year of 4 or 5 years. The average policyholder participation coefficient β is near to one and fixed fees are set at 100 basis points.
The life liabilities are backed by a portfolio of government and corporate bonds with fixed, or floating rate, which is constituted of a mix of government bonds issued by Italy, or Germany (80% Italian and 10% German), and corporate bonds with different credit ratings (7% BBB and 3% A or higher). The assets and liabilities have a modified duration of about 8 and 7 years, respectively. The asset portfolio has an operating (current) accounting return higher than 3% over the next 5 years. This features are typical of a traditional insurance product with a conservative investment profile.
Finally the unrealised gains (the difference between market and book value) on assets are about 15%, a very high level but not so unlikely for this kind of insurance product in Italy. This considerable amount of unrealised gains is due to the relentless compression of financial returns in the European Union.

[18]The relevance of the Italian traditional with profit as an example is explained in [Gambaro et al., 2017]. The same paper includes more information on the certainty equivalent and the market consistent approach to insurance valuations, which we propose.

5.2 Numerical results

We perform a Monte Carlo simulation using the dedicated apparatus we have developed and a set of stochastic scenarios consisting of the risk free rate, the Italian, German credit and liquidity spreads, and the corporate rating transition process. We haven't included the corporate sector spreads to reduce the calculation time. All the simulations have been conducted on retail laptops. The stochastic model is the one described in Section 3, with the calibration parameters reported in Tables 1, 2 and 3 in Section 4.
Table 4 reports the values of the VOG using our market consistent model, which includes all available stochastic risk factors (MCM), and the values of the VOG using the certainty equivalent approach (CEQ). It is remarkable the fact that the two approaches provide such different results. In particular while the CEQ approach seems to be more conservative when the return on assets is calculated using statutory accounting (LGAAP), the opposite is true when a mark to market performance is calculated (Mkt). We test two different investment strategies: Buy&Hold, which conventionally is the mostly adopted by investment managers and a constant mix one. The second strategy should increase the turnover of the portfolio incrementing the number of times assets are re-balanced. By doing so we expect an increase of the option value under LGAAP performance. Contrary to intuition[19] VOG diminishes using the CEQ model. These are all signs of the bias induced by CEQ, which is a too simple (although computationally parsimonious) model to capture the impact of different fixed income investment strategies, including a different mix of sovereign and corporate issuers.

Buy & Hold	CEQ (LGAAP)	CEQ (Mkt)	MCM (LGAAP)	MCM (Mkt)
VOG	49.04	171.51	8.96	224.52
(Std. Err.)	(0.05)	(0.07)	(1.10)	(6.90)
Const. Mix	**CEQ (LGAAP)**	**CEQ (Mkt)**	**MCM (LGAAP)**	**MCM (Mkt)**
VOG	24.23	170.18	65.56	263.37
(Std. Err.)	(0.02)	(0.07)	(5.07)	(9.13)

Table 4: The table reports the Value of Options and Guarantees (VOG) in million of Euro calculated running 500 stochastic simulations using the certainty equivalent (CEQ), or the market consistent model (MCM) we have developed. For each model we have calculated the VOG under local generally accepted accounting principle (LGAAP), or mark to market (Mkt). The investment strategy adopted are Buy&Hold and Constant Mix.

6 Sensitivity analysis

In this section we propose a sensitivity analysis, with the purpose of assessing the calibration, i.e. the risk that the statistical uncertainty in the parameter estimation can influence the economic valuation.
In classical calibration procedures, the estimated parameters are considered fixed values, instead we take into account the distribution of parameters estimator. The possibility

[19] A consequence of an higher turnover on the assets portfolio is the reduction of unrealised gains (or losses), which are used by insurance company to steer the performance credited to and shared with the policyholder.

to stress parameters such as correlation is important for a model to be of use as a risk management tool [20].

For the Vasicek model, we assume a multivariate normal distribution for the parameter estimator with a covariance matrix obtained as the inverse of the Fisher information matrix. For the Cox-Ingersoll and Ross model, we build empirical density histogram of parameters estimator, as already introduced in Section 4, using the bootstrap method proposed in [Efron and Tibshirani, 1986]. Figures 9, 10 and 11 show marginal density histograms for CIR parameters calibrated on Italian CDS data, German CDS data and corporate CDS index, respectively. The histograms are related to maximum likelihood calibrations whose results are presented in Tables 2 and 3.

Once we consider a probability distribution for model parameters, also the price (or in our example the VOG) turns out to be a random variable. Therefore, we should apply to the VOG distribution the invariant probabilistic sensitivity analysis typically used in operational risk management and based on the distance between the joint and the conditional cumulative distribution functions (CDF), for instance the Kuiper's test proposed in [Baucells and Borgonovo, 2013] and explained in detail in Appendix C. However, in our case, reconstructing an histogram of the VOG distribution is impossible in practice due to the complexity of the valuation and the long computational times. Hence, we implement the sensitivity analysis of the parameters with respect to the variance of the underlying fund at different maturities. Once we have detected critical parameters, then we perform the VOG calculation using critical stressed parameter value.

Table 5 reports the importance measures calculated with the Kuiper's distance of the model parameters with respect to the fund variance at two different maturities, 5 years and 10 years. It is clear from the table that the parameters of the Italian credit spread are critical, in particular the mean reversion parameter b and the long mean parameter $\bar{s}$. Figure 12 compares the joint CDF and the conditional CDF for a critical parameter, as the Italian credit spread parameter $\bar{s}$ and for a not critical parameter, as the German credit spread parameter η.

Tables 6, 7 and 8 report an example of possible stressing analysis, the value of critical parameters is stressed two standard errors then the impact on the mean and variance of the simulated Italian credit spread is tested and finally the VOG is repriced in the stressed scenarios. In particular, as we expect, the variance of the simulated Italian credit spread is consistently affected by the parameter stressing, while the mean value remains substantially unchanged. Moreover, intuitively, an increment of the Italian credit spread variance increase the VOG, while a reduction of the spread variance decreases the VOG. Using a two standard deviations stress of parameters reported in Table 6, the VOG changes significantly as reported in Table 8.

[20] The approach presented in this paper is also in agreement with the prudent person principle. For more details please check [AIFIRM, 2016, Paragraph 6]

Risk free rate

5 years			10 years		
$\boldsymbol{\beta}_a$	$\boldsymbol{\beta}_{\bar{r}}$	$\boldsymbol{\beta}_\sigma$	$\boldsymbol{\beta}_a$	$\boldsymbol{\beta}_{\bar{r}}$	$\boldsymbol{\beta}_\sigma$
0.0715	0.0777	0.0768	0.0755	0.0849	0.0858

Sovereign credit spread

Country	5 years				10 years			
	$\boldsymbol{\beta}_b$	$\boldsymbol{\beta}_{\bar{s}}$	$\boldsymbol{\beta}_\eta$	$\boldsymbol{\beta}_\rho$	$\boldsymbol{\beta}_b$	$\boldsymbol{\beta}_{\bar{s}}$	$\boldsymbol{\beta}_\eta$	$\boldsymbol{\beta}_\rho$
ITA	**0.3767**	**0.4849**	**0.3122**	0.0435	**0.4820**	**0.6184**	**0.3113**	0.0599
GER	0.0638	0.0702	0.0429		0.0689	0.0731	0.0410	

Corporate credit spread

5 years			10 years		
$\boldsymbol{\beta}_b$	$\boldsymbol{\beta}_{\bar{\lambda}}$	$\boldsymbol{\beta}_\eta$	$\boldsymbol{\beta}_b$	$\boldsymbol{\beta}_{\bar{\lambda}}$	$\boldsymbol{\beta}_\eta$
0.0705	0.0333	0.0704	0.0716	0.0363	0.0756

Table 5: In this table are reported the importance measures of the models parameters with respect to the fund variance. Two different time horizons are considered, 5 and 10 years. The importance measures are between 0 and 1, greater is the importance measure, greater is the impact of the parameters on the fund variance.

Italian credit	b	$\bar{s}$	η
calibrated	0.5430 (0.0308)	0.0270 (0.0001)	0.1712 (0.0020)
stressed (plus)	0.6046	0.0272	0.1752
stressed (minus)	0.4814	0.0267	0.1671

Table 6: The table reports the stressed critical parameters, i.e. the calibrated parameters plus or minus two standard errors.

Mean 5-years ITA credit spreads

time (years)	calibrated	stressed (plus)	% variation	stressed (minus)	% variation
1	3.42%	3.40%	0.8%	3.41%	0.5%
5	5.50%	5.49%	0.2%	5.46%	0.7%
10	3.45%	3.40%	1.3%	3.41%	1.3%
15	3.69%	3.68%	0.1%	3.66%	0.7%
20	4.34%	4.39%	-1.1%	4.38%	-0.8%

Variance 5-years ITA credit spreads

time (years)	calibrated	stressed (plus)	% variation	stressed (minus)	% variation
1	0.77%	0.67%	-13%	0.79%	3%
5	0.98%	0.87%	-11%	1.01%	4%
10	1.01%	0.86%	-15%	1.02%	1%
15	0.95%	0.90%	-5%	0.97%	2%
20	0.95%	0.90%	-5%	1.02%	7%

Table 7: The table report the variations of the mean and the variance of the 5 years Italian credit spread at different time horizons for stressed model parameters. Mean and variance are calculated using 500 scenarios.

MCM (LGAAP)

Buy&Hold	calibrated	stressed (plus)	difference	stressed (minus)	difference
VOG	8.96	5.23	-3.73	11.71	2.75
(Std. Err.)	(1.10)	(0.42)	-	(1.47)	-

Table 8: The table reports the VOG in million of Euro calculated running 500 stochastic simulations with calibrated and stressed critical parameters.

Conclusion

This paper proposes a simulative approach for the market consistent valuation of traditional with-profit life insurance funds able to deal with many sources of risk and their dependences. Our scenario generator specifies the risk factors dynamics under both the real world and the risk neutral probability measures. This approach attempt to reduce the distance between risk management and market consistent valuation in the spirit of the prudent person principle, such as the ones required under Solvency II in the insurance sector. To calibrate the risk factor models, we move from a "one trading day" cross section calibration to an approach which incorporates also historical information. Historical informations may be important for long-term, liability driven investments (like long-term savings and pension products) because it would introduce the possibility to calibrate more complex financial models, and eventually, to derive more meaningful stress testing strategies (sensitivity analysis), which are a fundamental tool for risk management. Obviously, this approach poses interesting challenges, in particular with respect to the definition of market price of risk and the assessment of calibration error in terms of the valuation risk induced, to which this paper provides some initial answers. Finally, the work presents an application of the scenario generator to the valuation of guarantees embedded in with-profit life insurance funds using different accounting rules and ALM strategies. Therefore, innovative sensitivity analyses are proposed in order to assess the valuation error.

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Figures

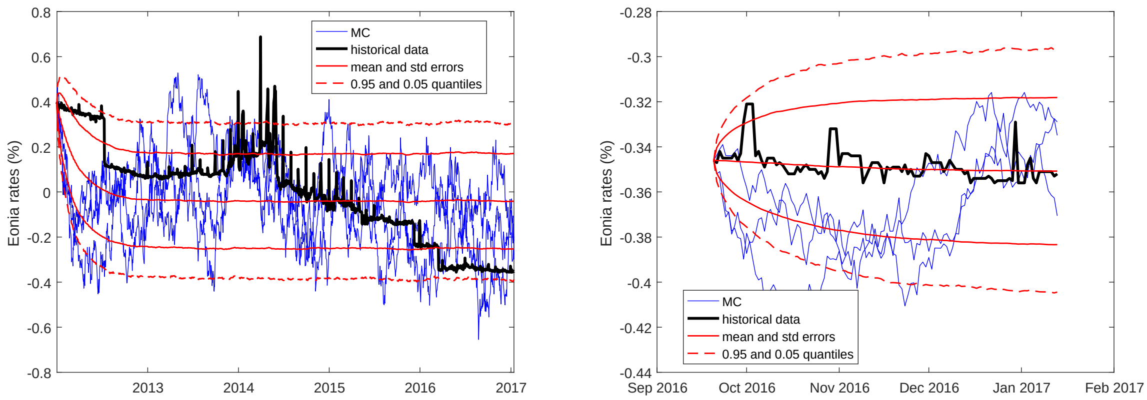


Figure 1: Historical calibration result on Eonia series from 2^{nd} January 2012 to 13^{th} January 2017 and from 20^{th} September 2016 to 13^{th} January 2017 of the Vasicek model (parameters in Table 1). The historical series is the black thick line, the yellow, green and blue lines are three simulated Monte Carlo paths. The continuous red lines represent the mean and the standard deviation (the mean plus or minus the standard deviation), the dashed red lines represent the 95% and the 0.5% quantiles. The mean, the standard deviation and quantiles are obtained with 10^4 Monte Carlo simulations.

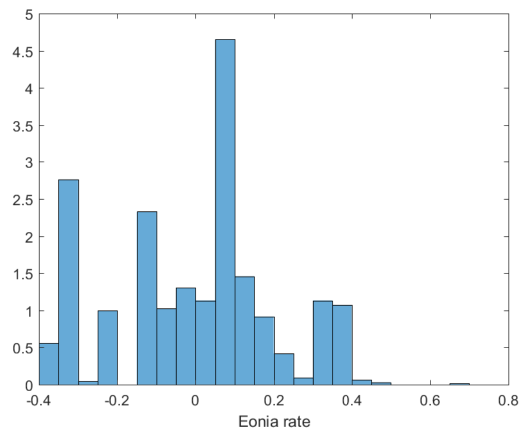
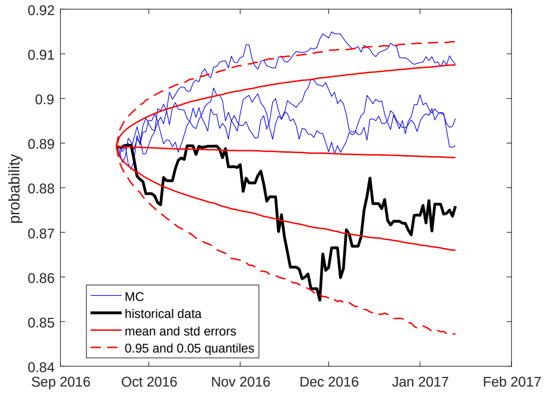
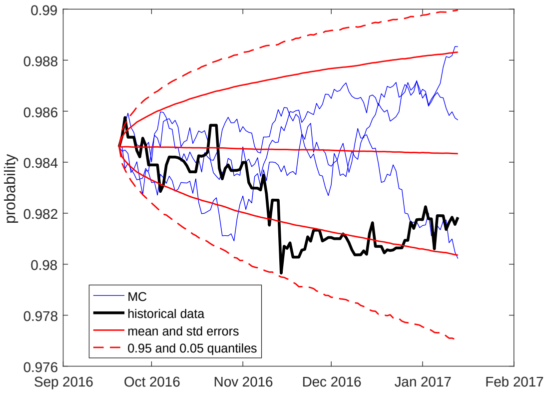


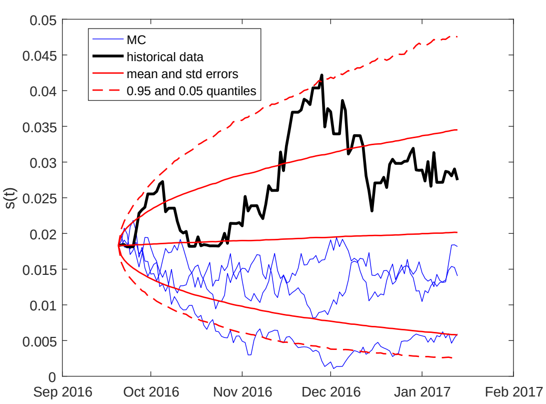
Figure 2: Histogram of the Eonia rate (in percentage) historical series from 2^{nd} January 2012 to 13^{th} January 2017.



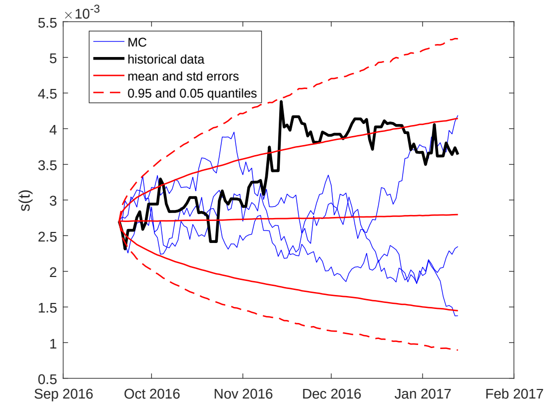
(a) Italian survival probability



(b) German survival probability

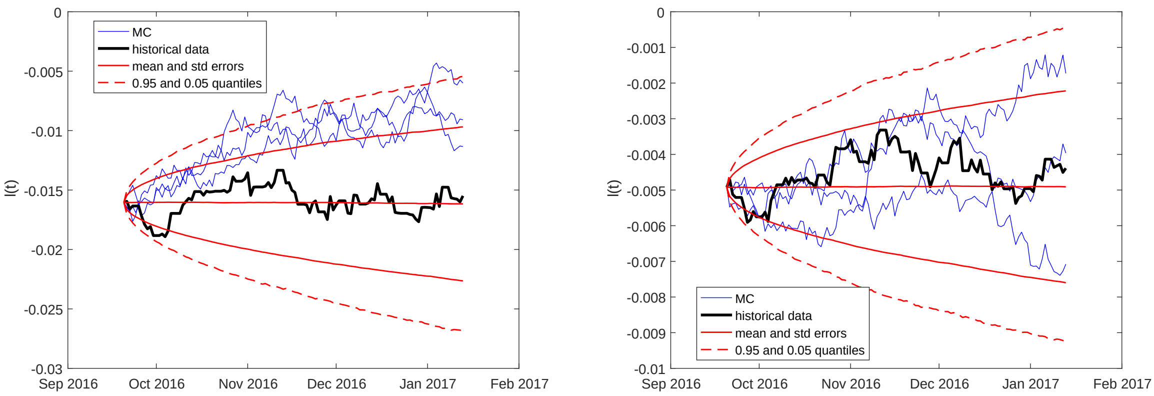


(c) Italian stochastic credit spread $s(t)$



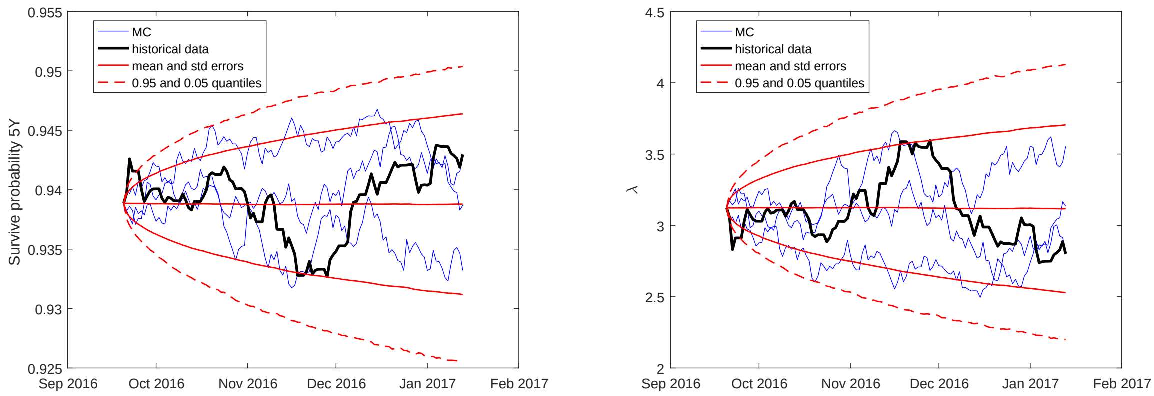
(d) German stochastic credit spread $s(t)$

Figure 3: Historical calibration results on 5Y Italian and German sovereign CDS from 20^{th} September 2016 to 13^{th} January 2017 of the CIR model (parameters in Table 2). The historical series is the black thick line, the yellow, green and blue lines are three simulated Monte Carlo paths. The continuous red lines represent the mean and the standard deviation (the mean plus or minus the standard deviation), the dashed red lines represent the 95% and the 0.5% quantiles. The mean, the standard deviation and quantiles are obtained with 10^4 Monte Carlo simulations.



(a) Italian stochastic liquidity spread $l(t)$ (b) German stochastic liquidity spread $l(t)$

Figure 4: Historical calibration results on 5Y Italian and German sovereign BVAL yield from 20^{th} September 2016 to 13^{th} January 2017 of the Ho-Lee model. The historical series is the black thick line, the yellow, green and blue lines are three simulated Monte Carlo paths. The continuous red lines represent the mean and the standard deviation (the mean plus or minus the standard deviation), the dashed red lines represent the 95% and the 0.5% quantiles. The mean, the standard deviation and quantiles are obtained with 10^4 Monte Carlo simulations.



(a) Survive probability (b) Stochastic intensity $\lambda(t)$

Figure 5: Historical calibration results on 5Y Itraxx Europe CDS Index S26 series from 20^{th} September 2016 to 13^{th} January 2017 of the CIR model. The historical series is the black thick line, the yellow, green and blue lines are three simulated Monte Carlo paths. The continuous red lines represent the mean and the standard deviation (the mean plus or minus the standard deviation), the dashed red lines represent the 95% and the 0.5% quantiles. The mean, the standard deviation and quantiles are obtained with 10^4 Monte Carlo simulations and parameters in Table 3.

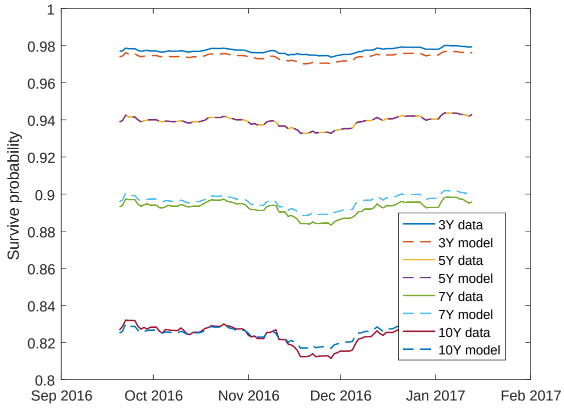


Figure 6: Historical calibration results on 5Y Itraxx Europe CDS Index S26 series from 20^{th} September 2016 to 13^{th} January 2017 of the CIR model. Continuous lines are market survival probabilities extracted from the series Itraxx Europe CDS Index S26 with maturities of 3Y, 5Y, 7Y and 10Y. The dashed lines represent the theoretical survival probabilities obtained using a stochastic intensity model for the transition matrix and parameters in Table 3.

Figure 7: Six months rolling correlations of historical series of industrial, financial and communication spreads from August 2009 to January 2017. The historical series of the spreads are obtained from Bloomberg sector bond index quotations (tickers BERCIN, BERCFIN and BERCCO).

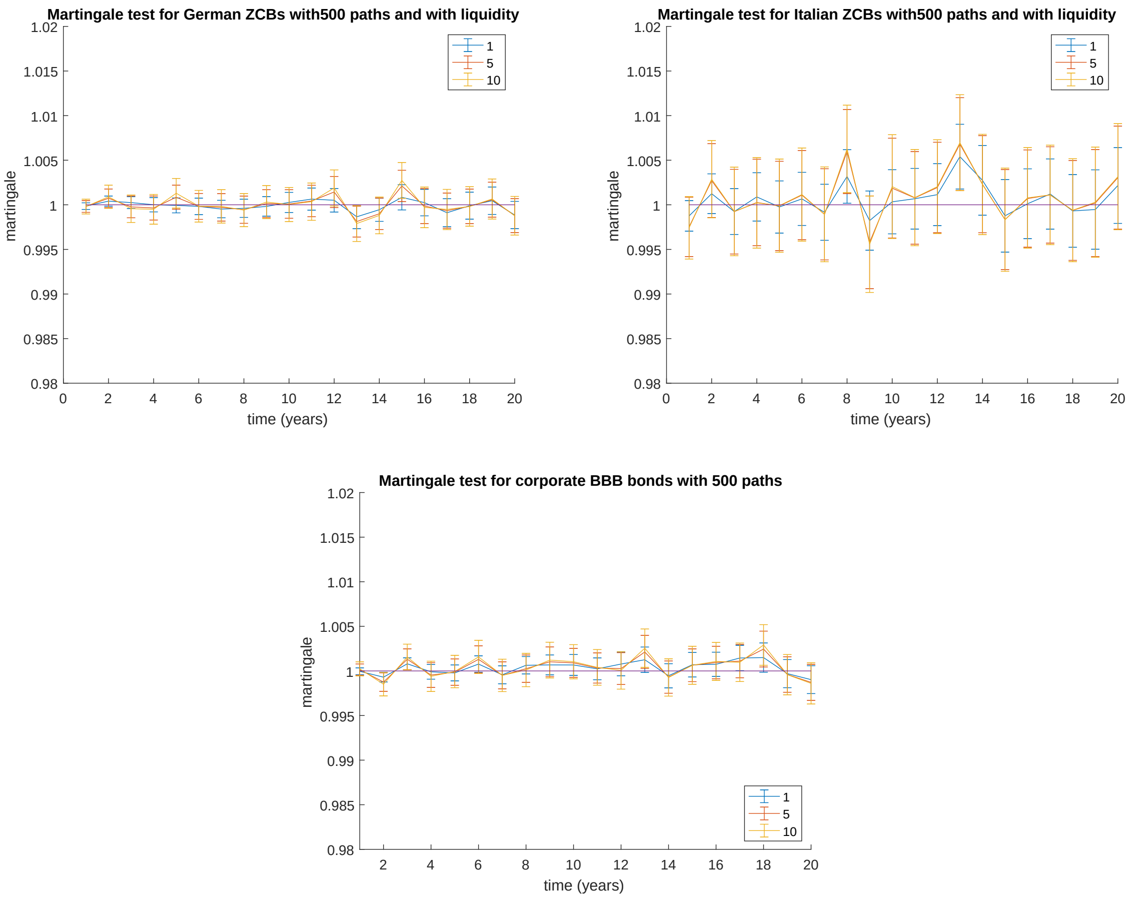
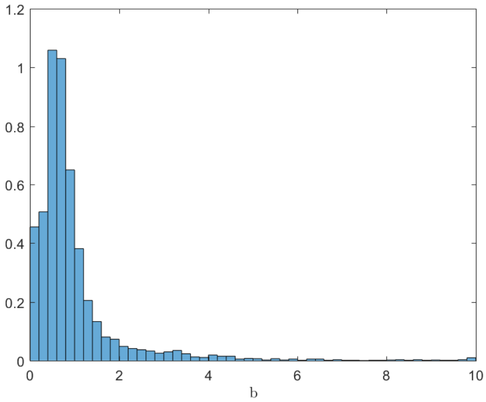
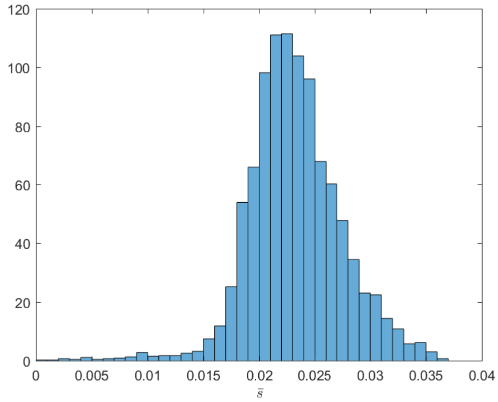


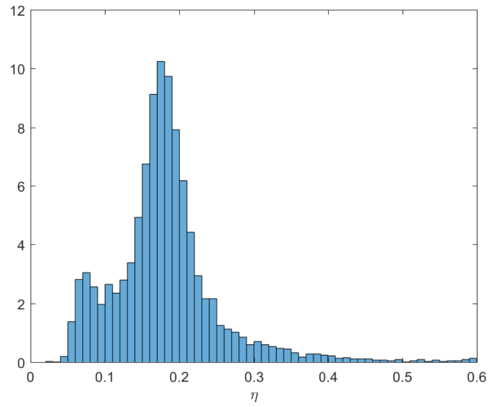
Figure 8: Martingale tests performed on sovereign German and Italian zero coupon bonds and corporate BBB bonds with 500 Monte Carlo simulations. The error bars are the 97.5% confidence intervals.



(a) Histogram of parameter b

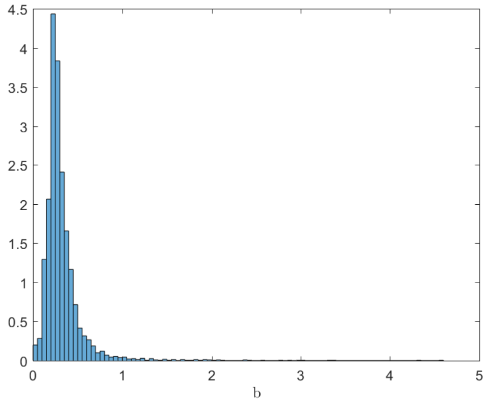


(b) Histogram of parameter $\bar{\lambda}$

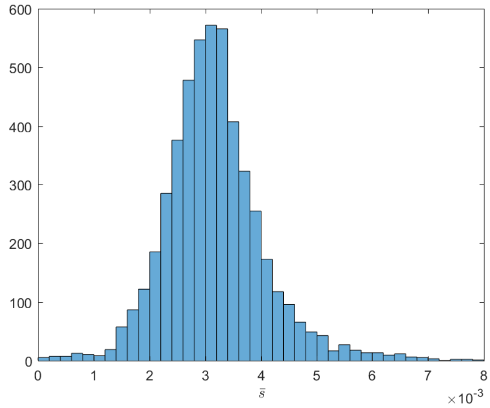


(c) Histogram of parameter η

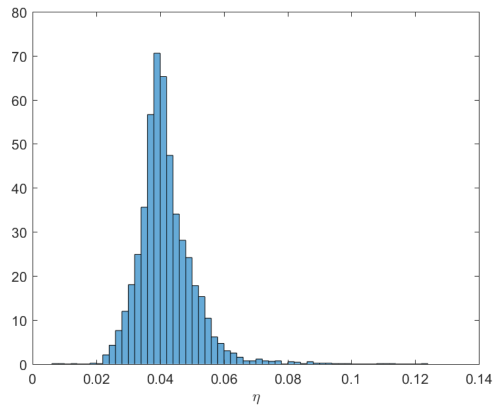
Figure 9: Histograms refer to Italian parameters estimated in Table 2.



(a) Histogram of parameter b

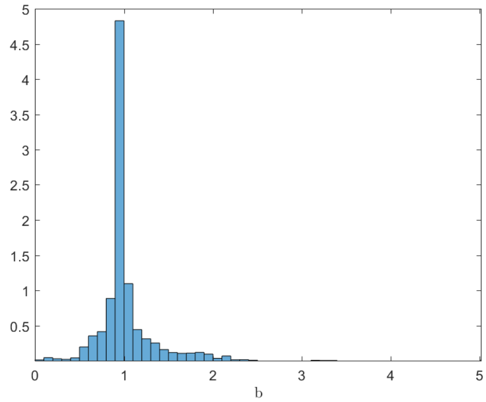


(b) Histogram of parameter $\bar{\lambda}$

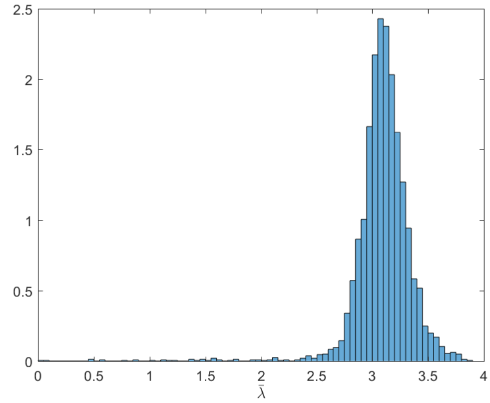


(c) Histogram of parameter η

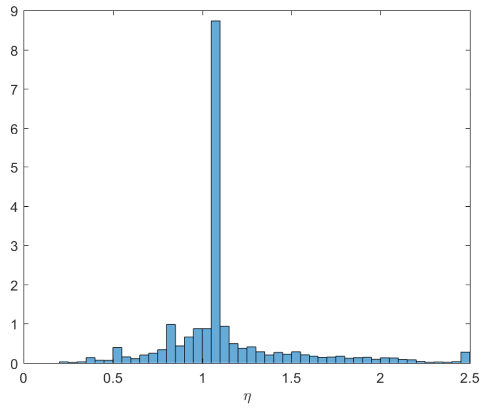
Figure 10: Histograms refer to German parameters estimated in Table 2.



(a) Histogram of parameter b

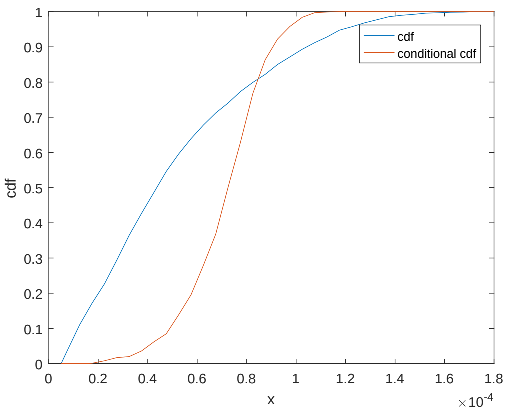


(b) Histogram of parameter $\bar{\lambda}$

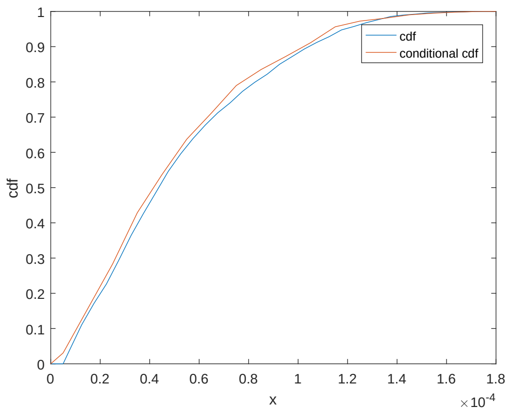


(c) Histogram of parameter η

Figure 11: Histograms refer to parameters estimated in Table 3.



(a) Italian parameter $\bar{s}$



(b) German parameter η

Figure 12: The two plots compare the joint and the conditional CDF of the volatility of the fund. For computing the conditional CDFs, the specified parameter is fixed equal to the value in Table 2.

A Maximum likelihood for default probabilities

In this section, we detail the procedure used to calibrate the CIR parameters via maximum likelihood method for the historical series of corporate default probabilities. The same procedure is applied to historical series of sovereign probabilities. Let θ be the set of the model parameters, we define

$$f(\lambda;\theta) := P(t,T)_{iK}, \tag{A.1}$$

and we assume that the function f is invertible with respect to $\lambda = \lambda(t)$. Once we fix a maturity τ and an initial rate i, then we can build the historical series of default probabilities $\{P_1, ..., P_N\}$ where $P_n = P(t_n, t_n + \tau)_{iK}$. The likelihood function of the observed default probabilities is

$$L(P_1, ..., P_N|\theta) = \prod_{n=1}^{N} h_P(P_n;\theta) \tag{A.2}$$

where $h_P(x;\theta)$ is the conditional density function of the default probabilities (we assume that $P_1, ..., P_N$ are i.i.d.). The default probability density function can be obtained using the density function of λ, which is a non-central chi-squared distribution, in the following way

$$\begin{aligned} h_P(P_n;\theta) &= h_\lambda(\lambda_n^*;\theta)\,\frac{\partial f^{-1}}{\partial P}(P_n;\theta) \\ &= h_\lambda(\lambda_n^*;\theta)\,\left(\frac{\partial f}{\partial \lambda}(\lambda_n^*;\theta)\right)^{-1} \end{aligned}$$

where $\lambda_n^* = f^{-1}(P_n,\theta)$. Hence, a maximum likelihood estimator of the parameters is

$$\hat{\theta} = \arg\max_\theta L(P_1, ..., P_N|\theta) = \arg\max_\theta \prod_{n=1}^{N} h_\lambda(\lambda_n^*;\theta)\,\left(\frac{\partial f}{\partial \lambda}(\lambda_n^*;\theta)\right)^{-1}.$$

B Change of measure derivation

The dynamic of $r(t)$ under the real world $\mathbb{P}$ measure follows a Vasicek dynamic as in section 3.3 equation (3.12)

$$dr(t) = a\,(\bar{r} - r(t))dt + \sigma\, dW^{\mathbb{P}}(t),$$

where $W^{\mathbb{P}}$ is a Brownian motion under the measure $\mathbb{P}$. We define the Radon-Nykodim derivatives from the real world measure $\mathbb{P}$ to the risk neutral measure $\mathbb{Q}$ as

$$\frac{d\,\mathbb{Q}}{d\,\mathbb{P}}(t) = e^{-\frac{1}{2}\int_0^t m^2(s)ds + \int_0^t m(s)dW^{\mathbb{P}}(s)}$$

where $m(t)$ is the market price, which has the following form

$$m(t) = \frac{\theta(t) - a\bar{r}}{\sigma},$$

with $\theta(t)$ a deterministic function of time.

By Girsanow theorem, the short rate $r(t)$ has the following dynamic under the risk neutral $\mathbb{Q}$ measure

$$dr(t) \;=\; (\theta(t) - ar(t))dt + \sigma dW(t),$$

where $W(t)$ is a Brownian motion under $\mathbb{Q}$. The previous process is an Hull-White process and it can be written as in equation (3.1) for opportunely chosen functions $\alpha(t)$ and $\theta(t)$ (see for instance [Brigo and Mercurio, 2006] page 72-73).
As in Section 3 the default risk of sovereign issuers is modelled using an intensity model (also called Cox processes or doubling stochastic Poisson processes) with intensities modelled using correlated CIR processes, for $i = 1, 2, ..., I$,

$$\begin{aligned}
\tau_i &= \inf\{t \geq 0 : \int_0^t \lambda_i(s)ds > \xi_i\}, \\
dy_i(t) &= b_i\left(\bar{s}_i - y_i(t)\right)dt + \eta_i\sqrt{y_i(t)}\,dZ_i^{\mathbb{P}}(t), \\
dZ_i^{\mathbb{P}}(t)\,dZ_j^{\mathbb{P}}(t) &= \rho_{ij}\,dt,
\end{aligned}$$

with I is the number of sovereign bond issuers, τ_i is the stochastic time of default of the i-th issuer, ξ_i are i.i.d. unitary exponential random variables and $Z^{\mathbb{P}}(t)$ ia a I-dimensional Brownian motions under $\mathbb{P}$.
By Girsanov theorem for point processes ([Bremaud, 1981]), defining the Radon-Nykodim derivatives as

$$\frac{d\,\mathbb{Q}}{d\,\mathbb{P}}(t) = \prod_{i=1}^{I}\left(\prod_{n\geq 1}\left(1 + \frac{\psi_i(t)}{y_i(t)}\right)\mathrm{I}(\tau_i(n) \leq t)\right)e^{\int_0^t \psi_i(s)\,ds}.$$

with $\psi_i(t)$ deterministic functions of time, we obtain that the credit risk intensity of the i-th issuer under the risk neutral probability is

$$s_i(t) = \psi_i(t) + y_i(t),$$

as in equation (3.5).
By similar reasoning, using the point process Girsanov theorem, the dynamic of the corporate credit risk intensity changes from the real world process as in equation (3.13) to the risk neutral measure process as in equation (3.11).

C Invariant probabilistic sensitivity analysis

Let y be a function of the model parameters (for instance y can be the VOG)

$$y(\theta) : D \to \mathbb{R}.$$

with $\theta \in D \subseteq \mathbb{R}^n$ and n is the number of the model parameters.
Let Θ be a random vector and θ is realization, hence $Y = y(\Theta)$ is the corresponding random model output and F_Y is the CDF of Y.

We define the importance measure of θ_i with respect to Y as

$$\beta_i = \mathbb{E}\left[d(F_Y, F_{Y|\Theta_i=\theta_i})\right],$$

where $0 < d < 1$ is a distance between the joint and the conditional CDF. We choose to use the Kuiper's metric, then

$$d(F_Y, F_{Y|\Theta_i=\theta_i}) = \sup_y\left(F_Y(y) - F_{Y|\Theta_i=\theta_i}(y)\right) + \sup_y\left(F_{Y|\Theta_i=\theta_i}(y) - F_Y(y)\right).$$

D A note on the use of interest rate Swaptions in insurance

A traditional with-profit (WP) life insurance policy is a contract designed to pay a lump sum at maturity or on death of the insured person, where the financial profit on the invested capital is shared between the insurance company and the policyholder. Typically these products offer a guarantee of minimum return on the capital invested so that it is normal to think about interest rate Swaptions[21] to hedge the guarantee against adverse market movements or in case the policyholder surrender its contract before maturity.

According to International Swap and Derivative Association at the end of 2013 the size of Swaptions market accounted for 12 trillions of dollars and these where mainly used to offset the volatility of funding-level (the difference between assets and liabilities) of Liability-Driven Investments such as pension funds and variable annuities[22] (see [International Swaps and Derivatives Association, Inc., 2014]). In fact, an investor seeking protection from declining interest rates could buy a receiver swaption[23] and exercise it in case swap rates fall below the strike, receiving in this way an higher rate than prevailing market.

Another application of this type of option in case of traditional WP life insurance products is to protect their funding-level from surrender by policyholders seeking better return from investment when interest rates rise[24].

Suppose for example that a policyholder every year, if interest rates rise above the return given by her investment, flips a coin and in case the result is tail, she asks her money back, while she keeps her money invested for another year if she gets a head. Suppose also that the insurance fund she is invested in, consists of a fixed coupon bond with a maturity equal to the investment's contractual maturity. The investment provides every year a statutory return[25] equal to the coupon of the bond, and this return is consolidated immediately after is determined in the insurance liabilities, so that in case the reimbursement is asked before contractual maturity, the insurance company has to pay to the policyholder the maximum between the market value of the bond and the consolidated amount (i.e. the capital investment plus the accrued performance). Suppose, for the sake of exemplification, that no management fee is charged to the policyholder.

In the first case (the result of coin flipping is tail) if the asset manager has hedged the investment with a receiver swaption (with a strike equal to the bond's coupon and a tenor equal to the residual contractual maturity), it won't face any loss when reimbursing the capital to the policyholder. If instead, the result is head, the investment manager would exercise the option (since it is in the money), sell the old bond, and with the proceeds buy a new par bond so that the next year performance will be aligned to the (higher) interest rate level. Finally, he would enter in a new swaption contract to protect the bond

[21] Swaptions are essentially an option to enter into an interest rate swap in the future

[22] Variable annuities come in a variety of flavours, each offering different types of guarantee and varying degrees of flexibility to the buyer. One of the most popular is the guaranteed minimum withdrawal benefit variable annuity, which allows the customer to withdraw guaranteed amounts on a regular, predetermined basis, regardless of the performance of the underlying assets – an instrument designed to provide retirement income protection

[23] A receiver swaption gives the purchaser the right to receive fixed in an interest rate swap; a payer swaption gives the buyer the right to pay a fixed rate

[24] The following example has been inspired by a similar one in [Harrison and Pliska, 1981]

[25] The performance of traditional insurance products is calculated according to accounting rules see [Gambaro et al., 2017] for the details.

value for another year.
Since this investment involves a rolling hedging strategy which is renewed every year if the policyholder doesn't surrender, the investment manger will find itself long Vega[26] because the performance of his strategy is subject to the cost of hedging. But the latter depends, ceteris paribus, to changes in option's volatility.
This example is less far from reality as it may seem at a first glance. Actually, a similar situation is faced by insurance companies when they do profit testing of new guaranteed products whit guarantees of Cliquet type. When the early termination of the contract is allowed at no cost, the insurance company faces a Vega risk which should be captured by a stochastic model of volatility. This has two relevant implications: If the model is incomplete (i.e. stochastic volatility is not modelled) pricing is definitely flawed. Next, simple financial model which can be calibrated on At The Money options won't be suitable for pricing this type of contracts and more complicated models should be adopted. However, since insurance contracts may have a long-term maturity, it may be a smart choice to consider also econometric models for the volatility modelling.
Alternatively, a easy and cheap solution is to pay the market value of the investment in case of early surrender. In fact, in this case, the pricing framework is complete even without stochastic volatility modelling, and simpler and easier (to calibrate) models, can be accommodated. Other options can limit the window where the policyholder can trigger the early termination of the contract to few years before the maturity, or limit the amount guaranteed in case of early termination.

[26]Vega measures the sensitivity of the value of an option to changes in the volatility of the underlying.